On the effect of wear on asperity heigth distributions, and the corresponding effect in the mechanical response

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Abstract

Since the time of the original Greenwood & Williamson paper, it was noticed that abrasion and wear lead to possibly bimodal distribution of asperity height distribution, with the upper tail of asperities following from the characteristics of the process. Using a limit case solution due to Borucki for the wear of an originally Gaussian distribution, it is shown here that the tail is indeed always Gaussian, but with different equivalent parameters. Therefore, if the wear process is light, one obtains a bimodal distribution and both may affect the resulting contact mechanics behaviour. In this short note, we illustrate just the main features of the problem. We conclude that it is an oversimplification to consider surfaces Gaussian.

1 Introduction

From the times of the celebrated Greenwood and Williamson (1966) paper, we know that surfaces that have been worn or abraded show a non-unique Gaussian distribution. Indeed, Fig.6 of GW (adapted here as Fig.1) shows in a Gaussian paper the distribution of summit heights in a surface of mild steel which had been abraded and then slid against copper, which resembles a "bimodal Gaussian". More precisely, Greenwood & Wu 2001 returned to that very surface and commented that Fig.6 of GW paper *"shows an abraded surface which is largely flat but where the heights of the upper 80% may be regarded as Gaussian. (We may note that 'proper' statistical tests would not reveal this fact)"*.

Fig.1 - Height distribution of an abraded surface (adapted from Greenwood and Williamson 1966).

On more careful consideration therefore, the surface had been largely worn out, but this may not be always the case. Today, we can gather a lot more insights into the process of wear or abrasion from the models developed, using GW description of roughness, in the context of Chemical Mechanical Polishing used for planarization of integrated circuits (Borucki, 2002, Borucki *et al.* 2004, Shi & Ring 2010). Indeed, polishing causes high asperities wearing faster than low asperities. Assume an original distribution of asperities, $\phi_0(z)$ which is taken to be Gaussian (but we postpone the term $\sqrt{2\pi}$ into the integrals for easy of notation)

$$\phi_0 = \frac{1}{\sigma} \exp\left[-\frac{z^2}{2\sigma^2}\right] \quad , \quad \overline{\phi}_0 = \exp\left[-\frac{z^2}{2}\right] \tag{1}$$

where σ is the rms amplitude of the summit heights.

Using Archard law for wear and the GW model, an equation similar to a differential Hamilton-Jacobi equation was obtained for the distribution of asperity heights which tends to develop very high peaks in the tail and indeed in the case when the separation between the polishing pad and the surface is kept constant at $z = c$, an analytical solution is developed with the method of

characteristics as (Borucki et al 2004, Shi & Ring 2010)

$$\phi(z) = \phi_0(z) \quad , \quad z < c \tag{2}$$

$$\phi(z) = \frac{w}{2\sqrt{z-c}}\left(t + \frac{2}{w}\sqrt{z-c}\right)\phi_0\left[c + \frac{w^2}{4}\left(t + \frac{2}{w}\sqrt{z-c}\right)^2\right] \quad , \quad z > c \tag{3}$$

$$= \frac{w}{2\sqrt{z-c}}\left(t + \frac{2}{w}\sqrt{z-c}\right)\phi_0\left[z + \frac{w^2}{4}t^2 + \frac{w}{2}t\sqrt{z-c}\right] \quad , \quad z > c \tag{4}$$

where $w = c_a \frac{4}{2\pi}\frac{E^*}{\sqrt{R}}$, with $c_a = kV$ and k is the wear constant, while V is sliding speed. Further, E^* is elastic modulus of materials, and R the radius of asperity summits. As Borucki et al (2004) clearly state:- "this solution develops an integrable singularity at $z = d$ representing the portion of the pad-surface that has been *worn smooth* by the wafer. After a sufficiently long but finite time, this singularity converges to a $\delta-$distribution with its amplitude approaching a limiting value given by the fraction of the pad-surface originally protruding above the wafer height". We need to take this carefully in consideration when developing a contact mechanics model. Before we do that, let us move to dimensionless form,

$$\overline{\phi}(\overline{z}) = \overline{\phi}_0 \quad , \quad \overline{z} < \overline{c} \tag{5}$$

$$\overline{\phi}(\overline{z}) = \left(\frac{\overline{t}}{2\sqrt{\overline{z}-\overline{c}}} + 1\right)\overline{\phi}_0\left[\overline{z} + \frac{\overline{t}^2}{4} + \overline{t}\sqrt{\overline{z}-\overline{c}}\right] \quad , \quad \overline{z} > \overline{c} \tag{6}$$

where we have introduced a dimensionless time

$$\overline{t} = wt/\sqrt{\sigma} \tag{7}$$

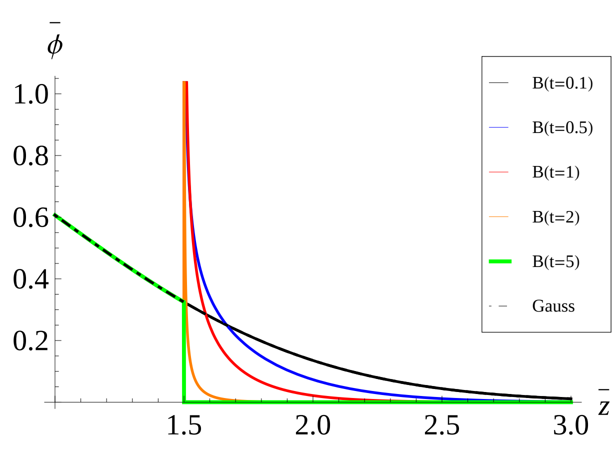

Fig.2. The distribution of asperity heights due to wear in Borucki's solution $\overline{\phi}_{\text{mod}}(\overline{z}, \overline{c}, \overline{t})$. $\overline{c} = 1.5$, and $\overline{t} = 0.1, 0.5, 1, 2, 5$. The delta function cannot be represented for obvious reasons.

This function is plotted in Fig.2 for representative dimensionless time $\overline{t} = 0.1, 0.5, 1, 2, 5$, and compared to the original Gaussian tail.

We can integrate analytically the difference between the Borucki distribution, and the original Gaussian one, to find the fraction of asperities worn out

$$F\left(\overline{c},\overline{t}\right) = -\int_{\overline{c}}^{\infty}\left(\overline{\phi}\left(\overline{z}\right)-\overline{\phi}_0\left(\overline{z}\right)\right)d\overline{z} = -\sqrt{\frac{\pi}{2}}\left[Erfc\left(\frac{\overline{c}+\overline{t}^2/4}{\sqrt{2}}\right)-Erfc\left(\frac{\overline{c}}{\sqrt{2}}\right)\right] \tag{8}$$

which we shall assume, in the absence of a better hypothesis, all lye on the $\overline{z} = \overline{c}$, and, perhaps an even stronger assumption, maintain the same radius. Obviously if special experimental setups will permit to have reasonably simple alternative assumptions, they could be readily incorporated.

Therefore, the so-modified Borucki distribution is

$$\overline{\phi}_{\text{mod}}\left(\overline{z},\overline{c},\overline{t}\right) = \overline{\phi}_0 \quad , \quad \overline{z} < \overline{c} \tag{9}$$

$$= \left(\frac{\overline{t}}{2\sqrt{\overline{z}-\overline{c}}}+1\right)\overline{\phi}_0\left[\overline{z}+\frac{\overline{t}^2}{4}+\overline{t}\sqrt{\overline{z}-\overline{c}}\right]+F\left(\overline{c},\overline{t}\right)\delta\left(\overline{z}-\overline{c}\right) \quad , \quad \overline{z} > \overline{c} \tag{10}$$

where δ is the classical delta function.

2 GW treatment

By developing the usual GW treatment for number of asperities in contact, area and load, integrating for the distribution of asperity heights, we obtain for the compression $d = (z_s - d_0)$

$$n = \frac{N}{\sqrt{2\pi}}\int_{d_0}^{\infty}\phi\left(z_s\right)dz_s = \frac{N}{\sqrt{2\pi}}\int_0^{\infty}\overline{\phi}\left(\overline{d}+\overline{d}_0\right)d\overline{d} \tag{11}$$

$$A = N\frac{\pi}{\sqrt{2\pi}}R\int_{d_0}^{\infty}\left(z_s-d_0\right)\phi\left(z_s\right)dz_s = \pi N R\sigma\int_0^{\infty}\overline{d\phi}\left(\overline{d}+\overline{d}_0\right)d\overline{d} \tag{12}$$

$$P = \frac{4}{3\sqrt{2\pi}}E^*NR^{1/2}\sigma^{3/2}\int_0^{\infty}\overline{d}^{3/2}\overline{\phi}\left(\overline{d}+\overline{d}_0\right)d\overline{d} \tag{13}$$

We can define the Gaussian case integrals as

$$I_n^g\left(\overline{d}_0\right) = \int_0^{\infty}\overline{d}^n\overline{\phi}_0\left(\overline{d}+\overline{d}_0\right)d\overline{d}$$

whereas the Borucki version "before the modification"

$$I_n^B\left(\overline{d}_0,\overline{c},\overline{t}\right) = \int_0^{\infty}\overline{d}^n\overline{\phi}\left(\overline{d}+\overline{d}_0,\overline{c},\overline{t}\right)d\overline{d}$$

and after the modification, in considering the $\overline{\phi}_{\mathrm{mod}}$ functions, additional contributions lead to Heaviside functions,

$$I^{B}_{0\,\mathrm{mod}}\left(\overline{d}_0,\overline{c},\overline{t}\right) = I^{B}_{0}\left(\overline{d}_0\right) + F\left(\overline{c},\overline{t}\right) H\left(\overline{c}-\overline{d}_0\right) \tag{14}$$

$$I^{B}_{1\,\mathrm{mod}}\left(\overline{d}_0,\overline{c},\overline{t}\right) = I^{B}_{1}\left(\overline{d}_0\right) + F\left(\overline{c},\overline{t}\right)\left(\overline{c}-\overline{d}_0\right) H\left(\overline{c}-\overline{d}_0\right) \tag{15}$$

$$I^{B}_{3/2\,\mathrm{mod}}\left(\overline{d}_0,\overline{c},\overline{t}\right) = I^{B}_{3/2}\left(\overline{d}_0\right) + F\left(\overline{c},\overline{t}\right)\left(\overline{c}-\overline{d}_0\right)^{3/2} H\left(\overline{c}-\overline{d}_0\right) \tag{16}$$

Further, writing

$$P^{g}_{0} = \frac{4}{3}\frac{N}{\sqrt{2\pi}}ER^{1/2}\sigma^{3/2}$$

and

$$A^{g}_{0} = \sqrt{\pi/2}\,(NR\sigma)$$

we can rewrite our final results as

$$n = \frac{N}{\sqrt{2\pi}}I^{B}_{0\,\mathrm{mod}}\left(\overline{d}_0,\overline{c},\overline{t}\right) \tag{17}$$

$$A = A^{g}_{0}I^{B}_{1\,\mathrm{mod}}\left(\overline{d}_0,\overline{c},\overline{t}\right) \tag{18}$$

$$P = P^{g}_{0}I^{B}_{3/2\,\mathrm{mod}}\left(\overline{d}_0,\overline{c},\overline{t}\right) \tag{19}$$

3 Results

The results are quite as expected. Fig.3 plots the integrals $I^{B}_{0\,\mathrm{mod}}, I^{B}_{1\,\mathrm{mod}}, I^{B}_{3/2\,\mathrm{mod}}$

for the same case of Fig.2. Starting with the integral giving the number of asperities in contact, it is clear that, at the "jump" $\overline{c} = 1.5$, we recover the number of asperities in the unworn profile, by construction. Upon increasing the wear time, the tail is worn out increasingly. Looking now at Fig.3b, this integral is proportional to the contact area, and, as in Fig.3c, a change of slope results in moving across the jump, but not an actual jump as in the number of asperities in Fig.3a.

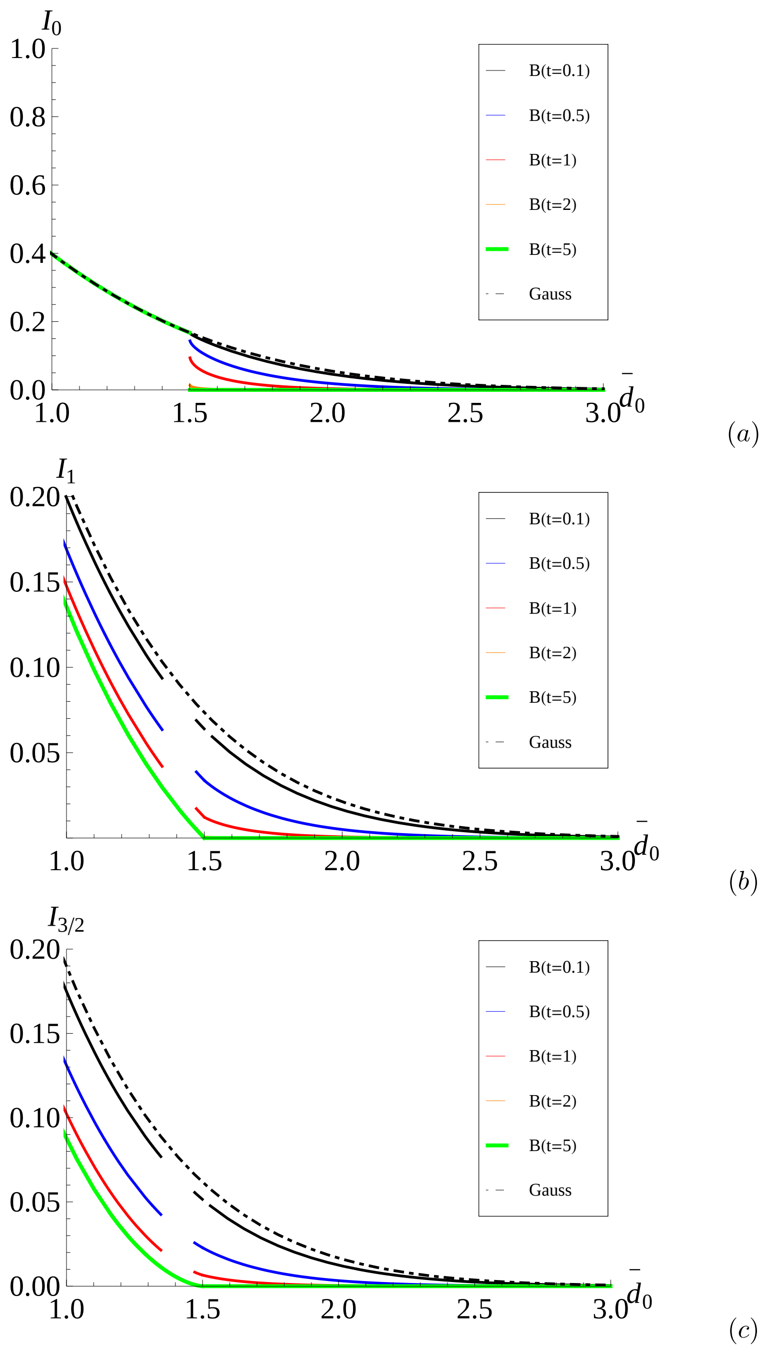

Fig.3. The integrals $I^B_{0\,\mathrm{mod}}, I^B_{1\,\mathrm{mod}}, I^B_{3/2\,\mathrm{mod}}$ (a,b,c, respectively). $\overline{c} = 1.5$, and $\overline{t} = 0.1, 0.5, 1, 2, 5$.

We can define a mean pressure on the contact

$$\overline{p}\left(\overline{d}_0,\overline{c},\overline{t}\right)=\frac{P\left(\overline{d}_0,\overline{c},\overline{t}\right)}{A\left(\overline{d}_0,\overline{c},\overline{t}\right)}=\frac{4}{3\pi}E\frac{\sigma^{1/2}}{R^{1/2}}\frac{I^B_{3/2\,\mathrm{mod}}\left(\overline{d}_0,\overline{c},\overline{t}\right)}{I^B_{1\,\mathrm{mod}}\left(\overline{d}_0,\overline{c},\overline{t}\right)} \tag{20}$$

which for the unworn given Gaussian system, would be relatively constant with separation. Fig.4 shows that the pressure is indeed more or less constant for any system, despite the constant changes with time, decreasing as long as the process continues – this is mainly due to the fact that the "effective" roughness is reduced in the tail of the distribution, because of wear. On the other hand, if the contact moves to separations lower than $\overline{c}=1.5$, then the mean pressure tends to restore rather quickly its original Gaussian value — it doesn't do immediately so because the asperities have now moved to a lower height, and therefore, with respect to the unworn case, they are compressed of a much lesser amount, resulting in less pressure. Obviously this suggests that for the procedure to really work at constant separation $\overline{c}$, the total load would need to decrease according to the model prediction.

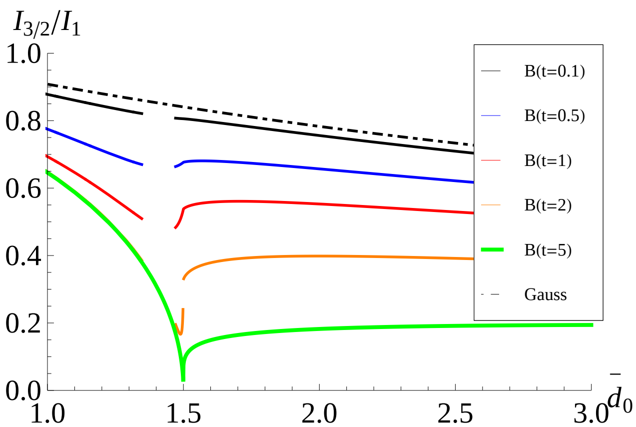

Fig.4. The ratio of the integrals $I^B_{3/2\,\mathrm{mod}}/I^B_{1\,\mathrm{mod}}$ which indicates mean pressure in the contact areas. $\overline{c}=1.5$, and $\overline{t}=0.1,0.5,1,2,5$.

4 Conclusions

We have shown in a simple case, that wear or abrasion leads to continuous change of roughness parameters, which affect the response of the rough contact. In cases when the process is selectively acting on part of the asperity distribution, the effects are clearer, and a bimodal distribution results. It may be therefore be an oversimplification to consider surfaces Gaussian, as if coming simply from a single distribution.

5 References

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